POLITECNICO DI MILANO

Distributed optimization over networks: application to multi-building energy management

Maria Prandini

Politecnico di Milano, Italy

maria.prandini@polimi.it

Credit

Alessandro Falsone

POLITECNICO MILANO 1863

Daniele Ioli

POLITECNICO MILANO 1863

Simone Garatti

POLITECNICO MILANO 1863

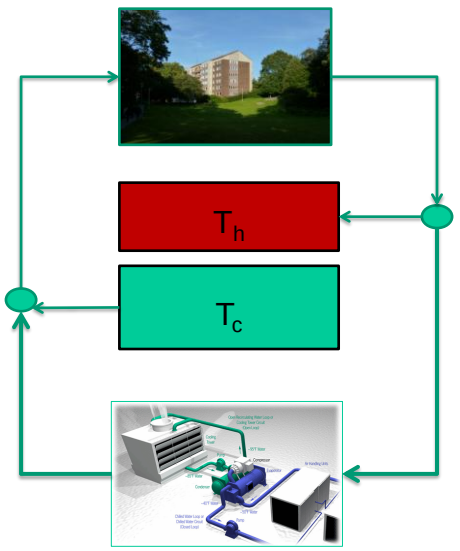
Kostas Margellos

UNIVERSITY OF OXFORD

Outline

- Building energy management:
  from a single building to a multi-building setup
- Distributed optimization over networks
- Distributed data-driven optimization over networks

Building cooling system with thermal storage

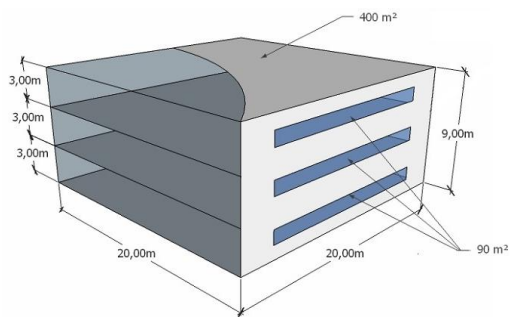


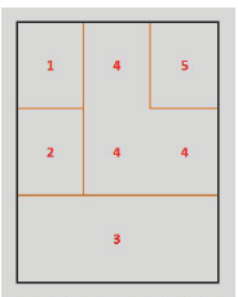
Building cooling system with thermal storage

We consider a setup that comprises

- a building composed of a number of thermally controlled zones

Building composed of multiple zones





Building cooling system with thermal storage

We consider a setup that comprises

- a building composed of a number of thermally controlled zones
- a chiller plant that converts the electrical energy in cooling energy
- a thermal storage unit that accumulates/releases cooling energy and hence shifts in time the cooling energy request to the chiller plant

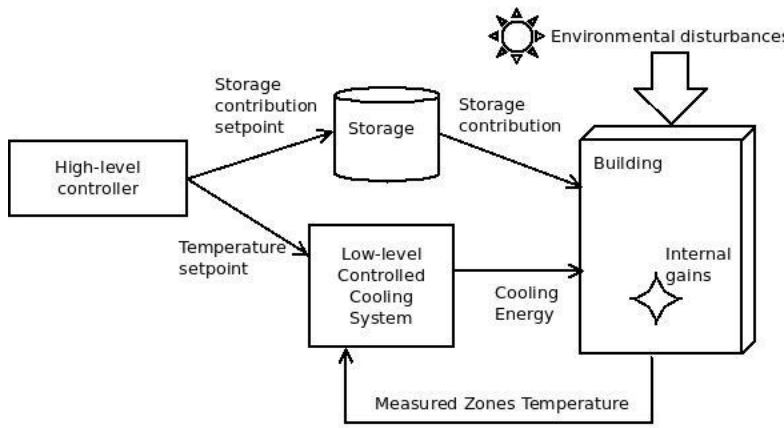
Energy management

Objective:

- operate the building cooling system with thermal storage so as to guarantee a certain comfort when occupants are present, while minimizing the electrical energy cost

Adopted approach

Act on the temperature set-points of the zones and on the storage energy exchange



Optimal energy management

Zone temperature set-points and storage charge/discharge command should be set appropriately in order to

- decrease the cooling power request
- exploit building thermal inertia to get an additional (passive) storage
- shift in time and set the cooling energy request to the chiller so as to use it at its maximal efficiency and request electrical energy to the grid when it is cheaper

while

- satisfying actuation constraints
- guaranteeing comfort conditions

Ingredients of the optimal control problem

- Cost function:
  electrical energy cost along some finite time-horizon
- Constraints:
  - comfort
  - actuation limits
- Control inputs:
  - zone temperature set-points *u*
  - thermal energy exchange with the storage *s*
- Disturbance inputs:
  - outdoor temperature
  - shortwave and longwave radiation
  - zone occupancy

Thermal energy balance

Cooling energy request to the chiller plant in each time slot *dt*:

$$E_{ch} = E_c - s$$

cooling load

energy exchange with the storage

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Corresponding electrical energy request :

$$E_\ell = \frac{a_1 T_o T_{cw} dt + a_2 (T_o - T_{cw}) dt + a_4 T_o E_{ch}}{T_{cw} - a_3 E_{ch}/dt} - E_{ch}$$

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outdoor temperature

temperature of the chilled water circuit, kept constant by low-level controller

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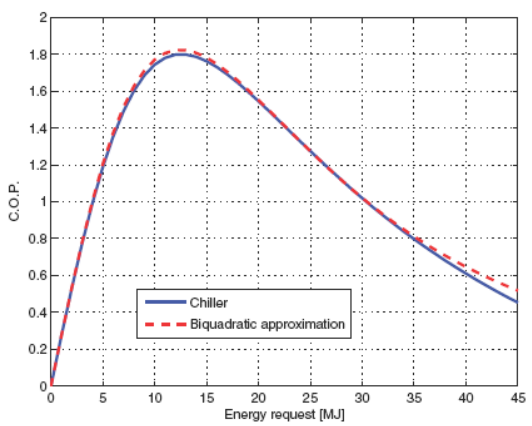
→ convex biquadratic approximation

$$E_\ell = c_1(T_o) E_{ch}^4 + c_2(T_o) E_{ch}^2 + c_3(T_o)$$

Efficiency of the chiller plant

Coefficient Of Performance:

$$\mathrm{COP} = \frac{E_{ch}}{E_l}$$



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$$E_\ell = c_1(T_o)E_{ch}^4 + c_2(T_o)E_{ch}^2 + c_3(T_o)$$

is convex in E_{ch}

Then, if E_{ch} linear in *u* e *s*

the cost function is convex in *u* e *s*

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Cooling load:

number of zones

$$E_c = \sum_{j=1}^{n_z} E_{c,j} = \sum_{j=1}^{n_z} (E_{w,j} + E_{p,j} + E_{\mathrm{int},j} + E_{z,j})$$

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energy exchange walls/zone

heat produced by people

heat produced by internal equipment, radiation through window

zone inertia

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Cooling load:

number of zones

linearly dependent on *u*, independent of *s*

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Constraints

- Comfort constraint:
  zone temperature set-point belongs to some interval that may depend on the time slot
  $\boldsymbol{u}_{min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{max}$

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Constraints are convex in *u* and *s*

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    $E_\ell \leq E_{max}$ [convex in *u* and *s*]

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    $E_\ell \leq E_{max}$ [convex in *u* and *s*]

Constraints are convex in *u* and *s*

Convex constrained optimization

The optimal energy management problem reduces to the following convex constrained optimization problem

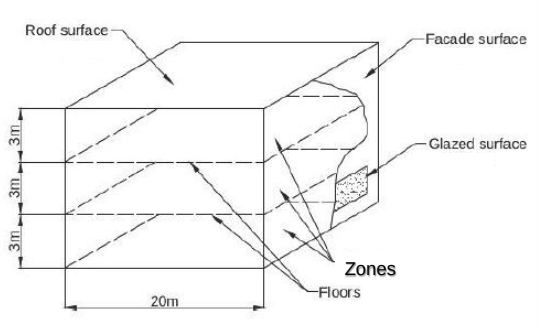
electrical energy price

$$\min_{\boldsymbol{u},\boldsymbol{s}} \quad \Psi \cdot \boldsymbol{E_\ell}(\boldsymbol{u},\boldsymbol{s})$$

$$\text{subject to: } \boldsymbol{u}_{min} \leq \boldsymbol{u} \leq \boldsymbol{u}_{max}, \quad |\boldsymbol{s}| \leq s_{max}$$
$$0 \leq \boldsymbol{S} \leq S_{max}, \quad \boldsymbol{E_c} \geq 0, \quad \boldsymbol{E_\ell}(\boldsymbol{u},\boldsymbol{s}) \leq E_{max}$$

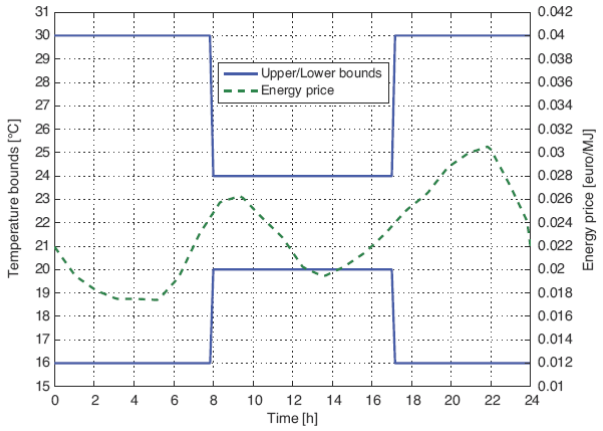
A numerical example

Building structure



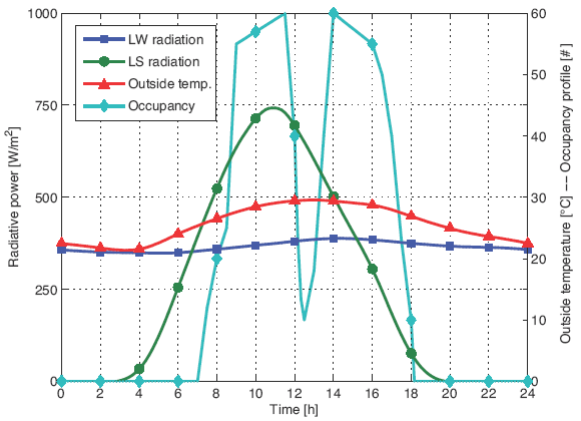
A numerical example

Comfort constraints and energy price along a 1 day time-horizon



A numerical example

Disturbances along a 1 day time-horizon



A numerical example

Look-ahead time horizon: 24 hours

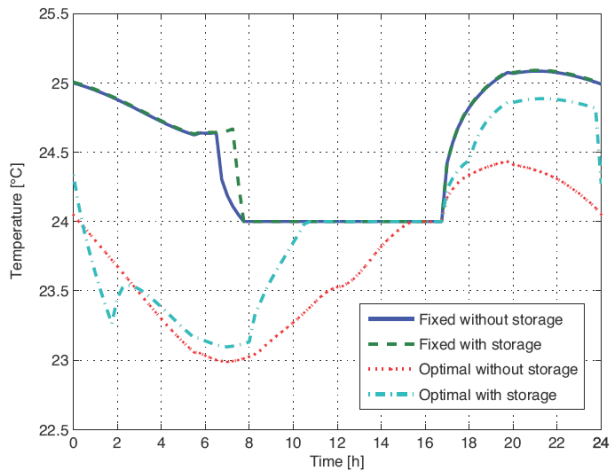
Time slot *dt*: 10 minutes

4 policies are compared:

- Fixed:
  temperature kept constant during working hours; chiller idle otherwise
  storage is charged at night and discharged during the day
- Optimal:
  solution of the constrained optimization problem over 48 hours
- Fixed without storage
- Optimal without storage

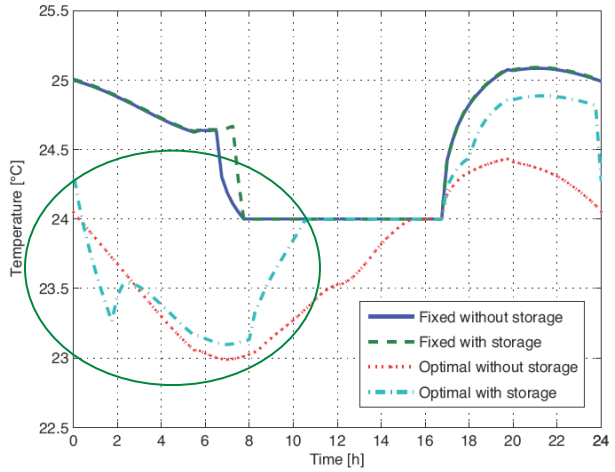
A numerical example: single zone

Zone temperature set-point



A numerical example: single zone

Pre-cooling phase to exploit the building as a passive storage



A numerical example: single zone

Strategy	F	F+S	O	O+S
E_c [MJ]	1094	1087	1288	1076
E_{ch} [MJ]	1094	1330	1288	1187
E_ℓ [MJ]	1219	742.3	750.5	694.4
Cost [euro]	29.09	16.95	16.63	14.44

A numerical example: single zone

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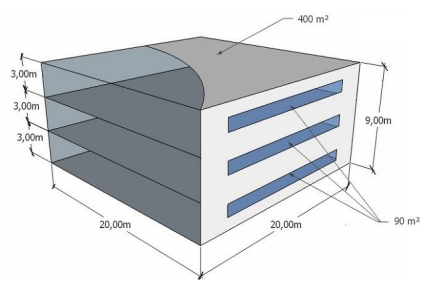
The chiller plant works at better efficiency levels

A numerical example: multiple zone setting

Zone temperature set-point is different for the three floors



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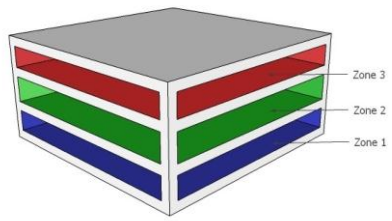
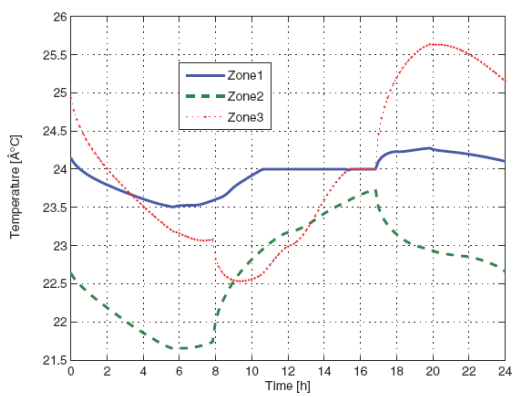


Optimal policy without storage is considered

A numerical example: multiple zone setting

Zone temperature set-points [optimal policy without storage]



The intermediate floor (zone 2) is used as a thermal storage which drains heat from other floors through its pavement and its ceiling

A numerical example: single vs multiple zone

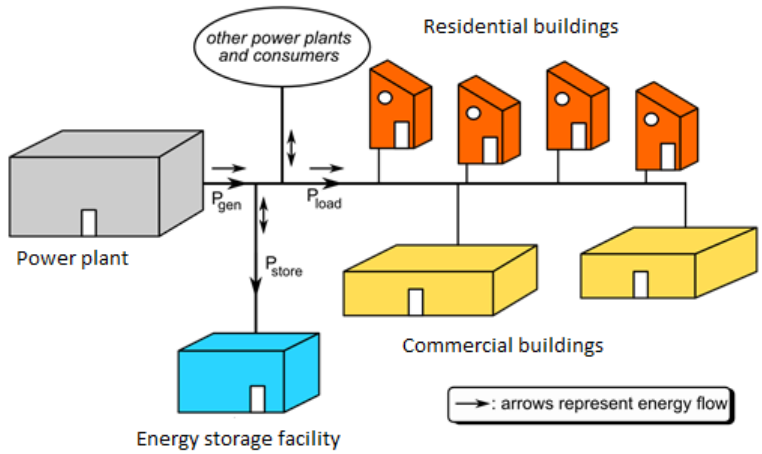
	Single-Zone	Multi-Zone	$\Delta_{\%}$
E_c [MJ]	1288	979.9	-23.9%
E_{ch} [MJ]	750.5	669.7	-14.5%
Cost [euro]	16.63	12.79	-21.1%

The added flexibility allows to better exploit the building inertia

Single building setup

- a convex formulation of the optimal energy management problem for a building cooling system with thermal storage was introduced
- the model is easily scalable in the number of zones, can be generalized to a multi-building setup

Multi-building setup



Multi-building setup

District network

- m buildings, possibly divided into zones
- a chiller unit for each building
- a single shared energy storage

- Control Inputs:
  zone temperatures
  storage energy exchange

- Disturbance inputs:
  - occupancy
  - outdoor temperature
  - shortwave and longwave radiation

Multi-building setup

District network

- m buildings, possibly divided into zones
- a chiller unit for each building
- a si

- Contro
  zone
  storag

- Disturb
  - oc
  - outdoor temperature
  - shortwave and longwave radiation

Issues

Computation:

Problem size too big!

Communication:

Not all communication links at place; link failures

Information privacy:

buildings may not want to share their consumption profiles

Outline

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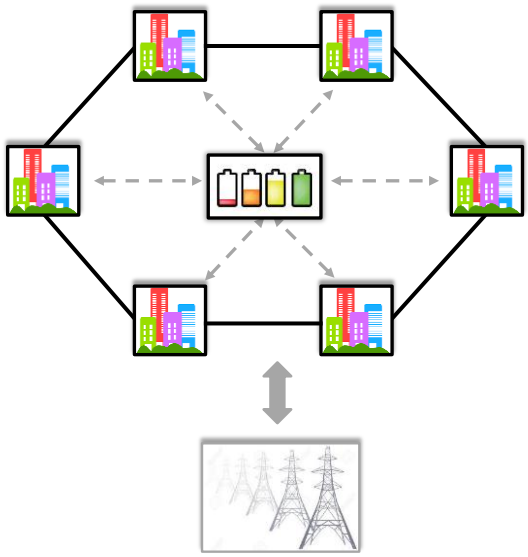
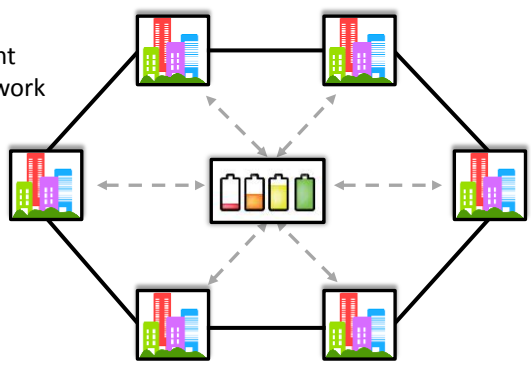
Problem setup



Problem setup

agent
network



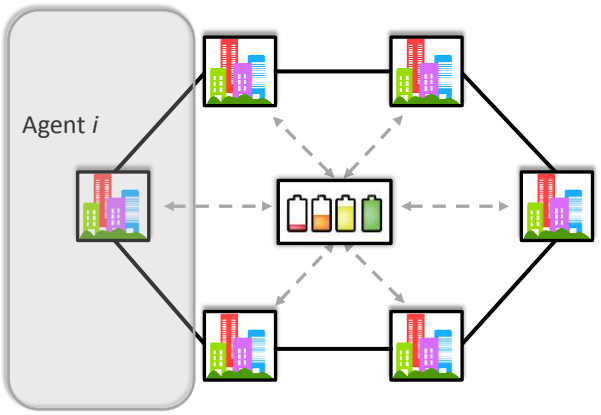
- network of $i = 1,2,\dots,m$ cooperative agents
- let x denote the global decision vector to be optimally agreed upon

Centralized optimization problem

$$\min_x \sum_{i=1}^{m} f_i(x)$$
$$\text{subject to } x \in \bigcap_{i=1}^{m} X_i$$

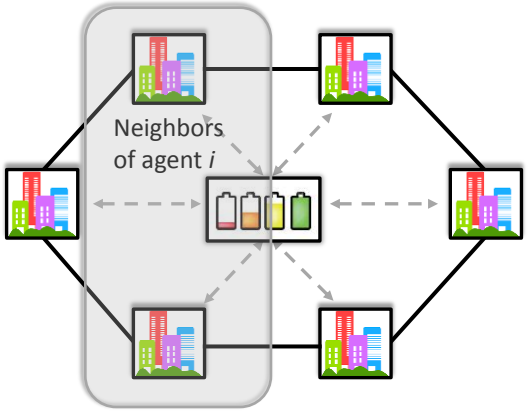
- $f_i(x)$: Objective/utility function of agent *i*
- X_i : Physical/technological constraints of agent *i*
- Coupling via common decision variables and constraints

Distributed architecture



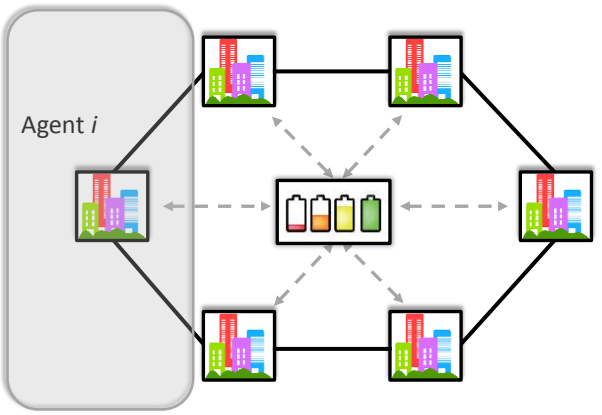
Step 1: agent *i* solves a local decision problem and makes a tentative (local) decision for *x*

Distributed architecture



Step 2: neighbouring agents communicate their tentative decisions to agent *i*

Distributed architecture



Step 3: Agent *i* weights the received information, solves a refined problem and makes a new decision for *x*

Distributed architecture in math

Initialization

$k = 0.$

Consider $x_i(0) \in X_i$, for all $i = 1, \ldots, m$.

For $i = 1, \ldots, m$ **repeat until convergence**

$z_i(k) = \sum_{j=1}^{m} a_j^i(k) x_j(k).$

$x_i(k+1) = \arg\min_{x_i \in X_i} f_i(x_i) + \frac{1}{2c(k)} ||z_i(k) - x_i||^2.$

$k \leftarrow k + 1.$

local objective

local constraint set

Distributed architecture in math

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local objective

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penalty from neighboring average

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Scalable method:

- communication only between neighbours
- computation only local, in parallel for agents

Information privacy:

- agents do not share information about their preferences/needs with the other agents

Convergence analysis

- $f_i(x)$: convex
- X_i : convex, compact

$$\min_{x_i \in X_i} f_i(x_i)$$

Convergence analysis

- $f_i(x)$: convex
- X_i : convex, compact

$$\min_{x_i \in X_i} f_i(x_i) + \frac{1}{2c(k)} ||z_i(k) - x_i||^2$$

Choice of the proxy term

Penalty coefficient $c(k)$ is positive, non-increasing, should not decrease too fast

$$\sum_{k=0}^{\infty} c(k) = \infty,$$
$$\sum_{k=0}^{\infty} c(k)^2 < \infty.$$

so that asymptotically we give emphasis to consensus, but without compromising the possibility to achieve optimality

Convergence analysis

Information mix

Weight coefficients in $z_i(k) = \sum_{j=1}^{m} a_j^i(k) x_j(k)$

- Satisfy a non-zero lower bound if link *i-j* is present
  → info mixing at a non-diminishing rate
- form a doubly stochastic matrix, i.e.

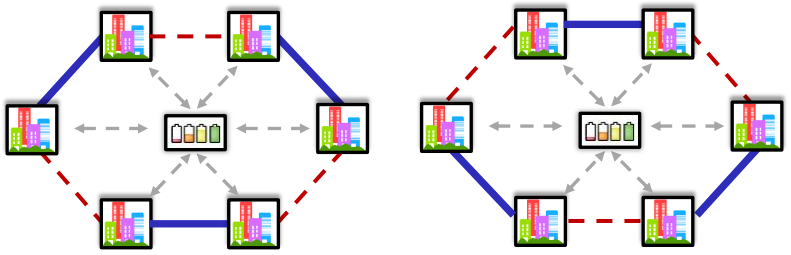
$$\sum_{i=1}^{m} a_j^i(k) = 1$$
$$\sum_{j=1}^{m} a_j^i(k) = 1$$

→ agents influence each other equally in the long run

Convergence analysis

Network connectivity

- Any pair of agents communicates infinitely often, possibly through a communication graph that changes through iterations



- The intercommunication time is bounded

Convergence analysis

- Consensus: agents' estimates converge to their arithmetic average

$$x_i(k) \xrightarrow[k\to\infty]{} \frac{1}{m}\sum_{i=1}^{m} x_i(k)$$

Convergence analysis

- Consensus: agents' estimates converge to their arithmetic average

$$x_i(k) \xrightarrow[k\to\infty]{} \frac{1}{m}\sum_{i=1}^{m} x_i(k)$$

- Optimality: asymptotic convergence to some minimizer of the centralized problem

$$x_i(k) \xrightarrow[k\to\infty]{} x^*$$

Numerical example – District definition

District configuration

3 identical buildings with three zones

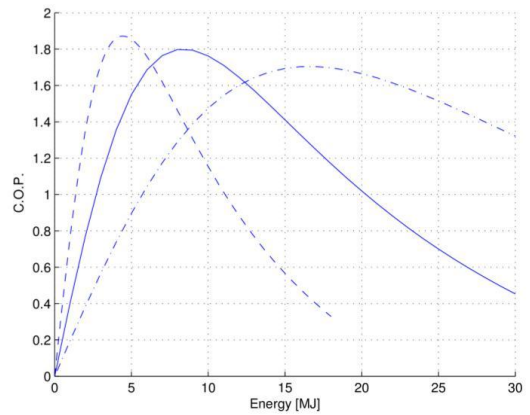
and with different chillers

Chiller types:

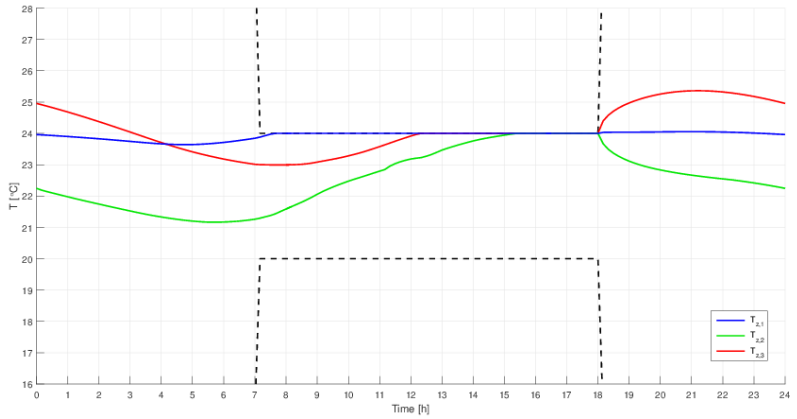
'small' → building 2

'medium' → building 1

'large' → building 3

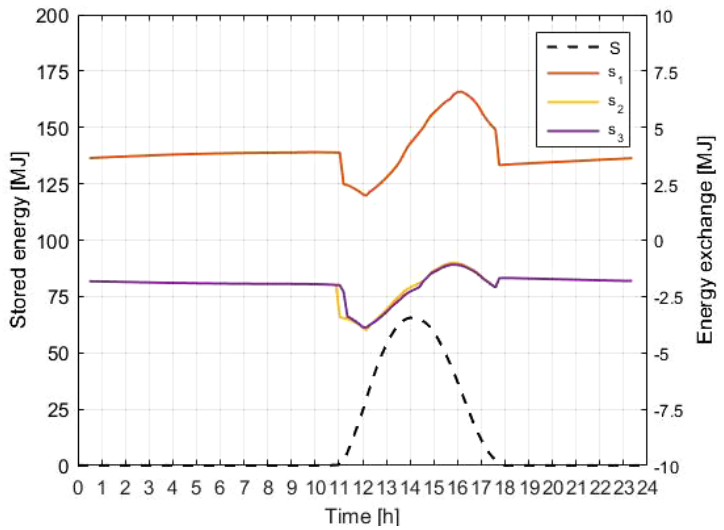


Results: zone temperature set-points



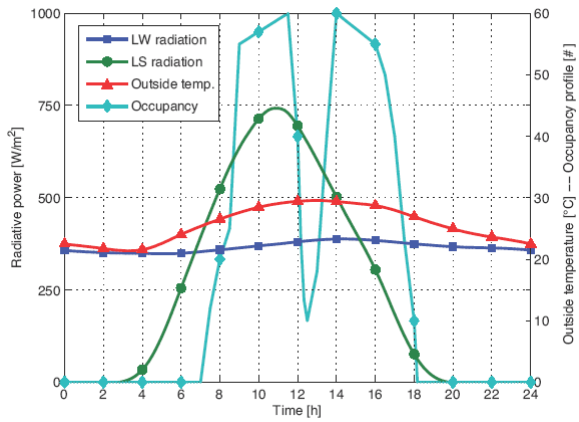
- Strong pre-cooling phase
- Middle floor (green line) used as additional thermal storage

Energy exchange with the storage: solution computed by building 1

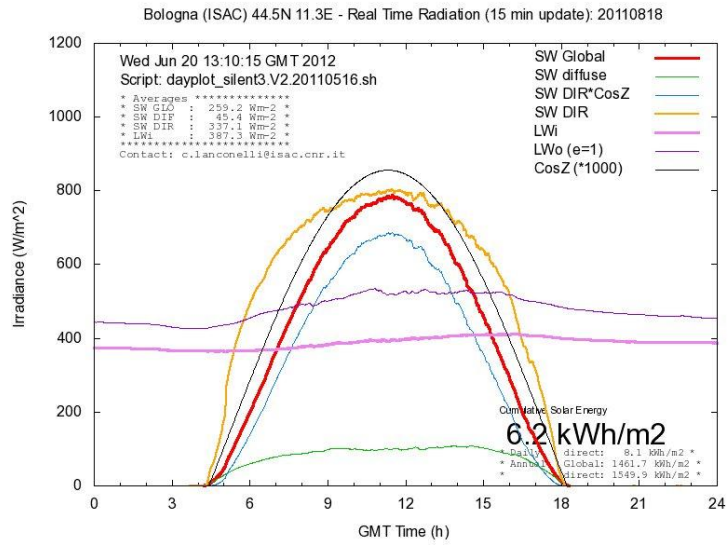


Uncertainty

Realizations of the disturbances along a 1 day time-horizon

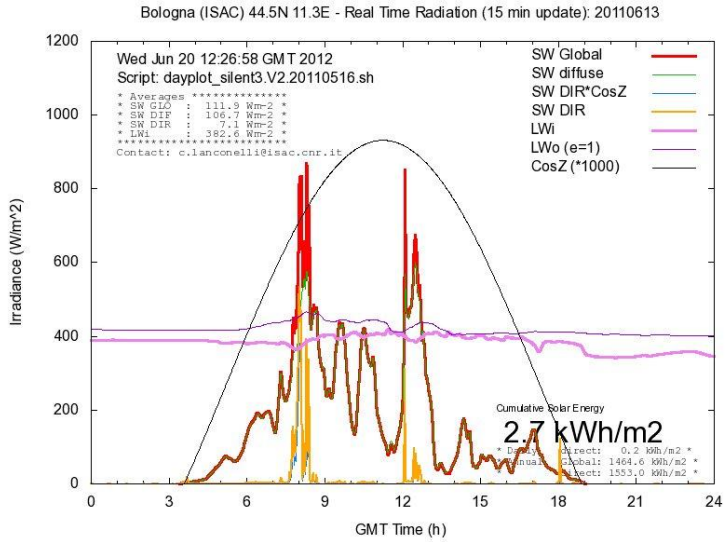


Solar radiation data



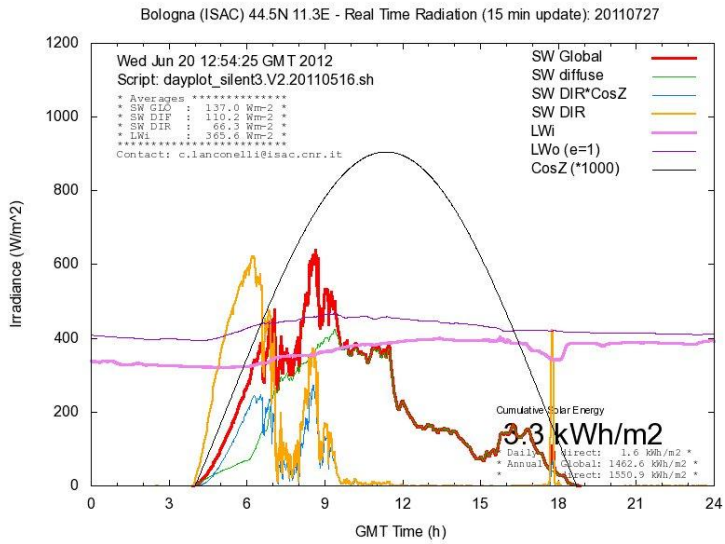
Courtesy of Istituto di Scienze dell'Atmosfera e del Clima (ISAC) - CNR

Solar radiation data



Courtesy of Istituto di Scienze dell'Atmosfera e del Clima (ISAC) - CNR

Solar radiation data



Courtesy of Istituto di Scienze dell'Atmosfera e del Clima (ISAC) - CNR

Outline

- Building energy management problem: from a single building to a multi-building setup
- Distributed optimization over networks
- Distributed data-driven optimization over networks

Centralized stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x)$$
$$\text{subject to } x \in \bigcap_{i=1}^{m} X_i$$

$$\boxed{X_i \longrightarrow X_i(\delta)}$$

- δ: uncertainty vector

Centralized stochastic optimization

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- δ: uncertainty vector
- uncertainty distributed on set Δ according to probability P

Centralized stochastic optimization

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$$\boxed{X_i \longrightarrow X_i(\delta)}$$

- δ: uncertainty vector
- uncertainty distributed on set Δ according to probability P
- only scenarios $\delta^{(1)}, \delta^{(2)}, \ldots, \delta^{(N)}$ available

Centralized stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \text{subject to } x \in \bigcap_{k=1}^{N} \bigcap_{i=1}^{m} X_i(\delta^{(k)}) \longrightarrow x_N^c$$

Centralized stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \text{subject to } x \in \bigcap_{k=1}^{N} \bigcap_{i=1}^{m} X_i(\delta^{(k)}) \longrightarrow x_N^c$$

how does the solution x_N^c generalizes to unseen scenarios?

Centralized stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \longrightarrow \quad x_N^c$$

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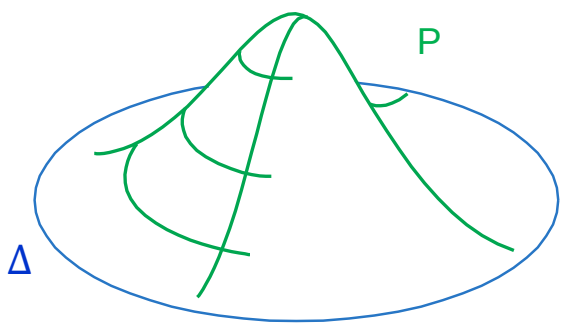
how does the solution x_N^c generalizes to unseen scenarios?

$$\mathbb{P}\left\{ \delta : x_N^c \notin \bigcap_{i=1}^{m} X_i(\delta) \right\}$$

Scenario approach

$$\min_{(\eta,h)} h$$

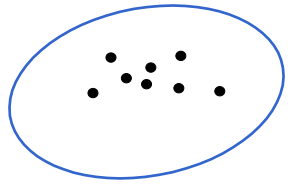
$$\text{subject to } l_\delta(\eta) \leq h, \forall \delta \in \Delta$$



Uncertainty set Δ endowed with a probability measure P

Scenario approach

Pick $\delta^{(1)}, \ \delta^{(2)}, \dots, \delta^{(N)}$ at random from Δ, according to P



Scenario approach

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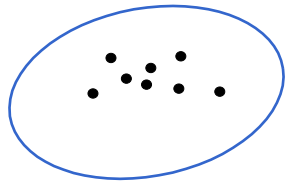


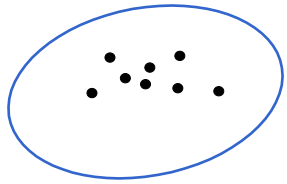
$$\min_{(\eta,h)} h$$

subject to $l_{\delta^{(i)}}(\eta) \leq h, i = 1, \dots, N$

Consider only a finite number N of constraints

Scenario approach

Pick $\delta^{(1)}, \ \delta^{(2)}, \dots, \delta^{(N)}$ at random from Δ, according to P



$$\min_{(\eta,h)} h$$

subject to $l_{\delta^{(i)}}(\eta) \leq h, i = 1, \dots, N$

Consider only a finite number N of constraints

how robust is the scenario solution $\theta_N = (\eta_N, h_N)$?

Scenario approach

$\{\delta: l_\delta(\eta_N) > h_N\}$ is the Violation Set of $\theta_N = (\eta_N, h_N)$

$V(\theta_N) = P\{\delta: l_\delta(\eta_N) > h_N\}$ is the Violation of $\theta_N = (\eta_N, h_N)$

Scenario approach

$\{\delta\colon l_\delta(\eta_N) > h_N\}$ is the Violation Set of $\theta_N = (\eta_N, h_N)$

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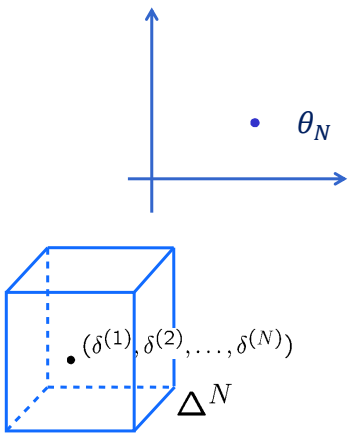

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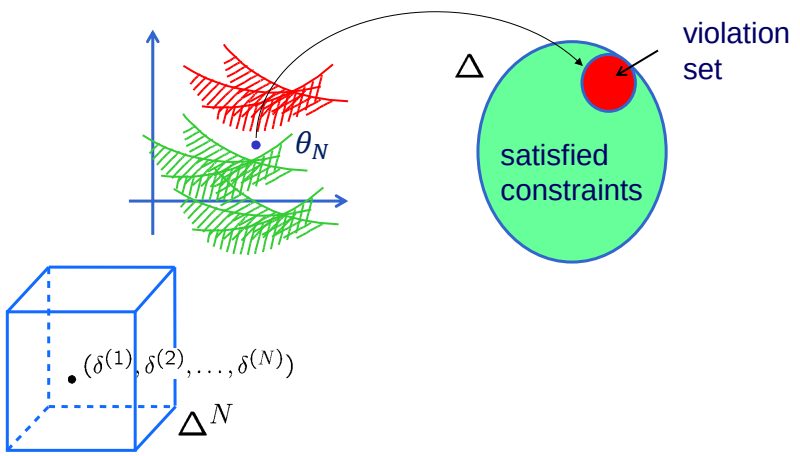

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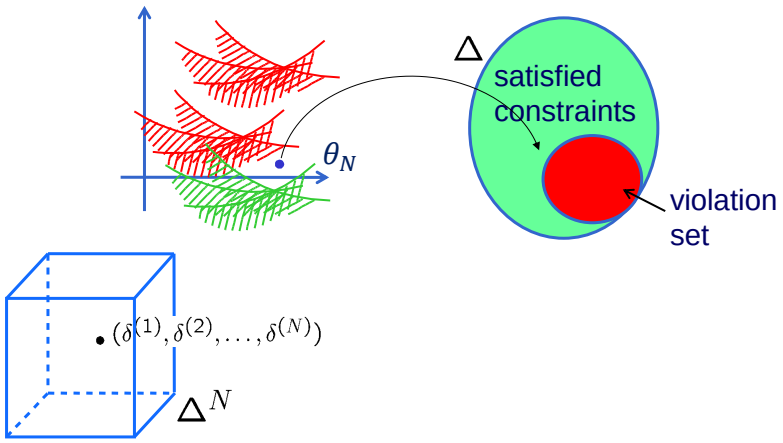

Scenario approach

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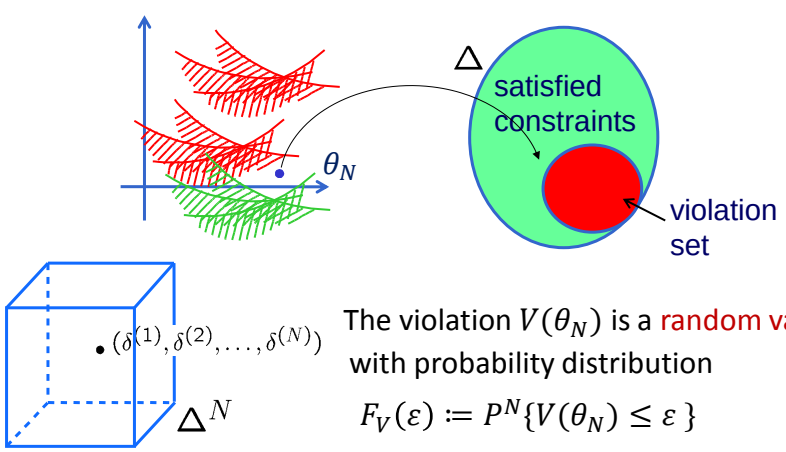
$V(\theta_N) = P\{\delta\colon l_\delta(\eta_N) > h_N\}$ is the Violation of $\theta_N = (\eta_N, h_N)$



The violation $V(\theta_N)$ is a random variable with probability distribution

$$F_V(\varepsilon) := P^N\{V(\theta_N) \leq \varepsilon\}$$

Scenario approach

$$\min_{(\eta,h)} h$$

subject to $l_{\delta^{(i)}}(\eta) \leq h, i = 1, \dots, N$

scenario optimization

Theorem

If $f_\delta(\theta) = l_\delta(\eta) - h$ is convex in $\theta = (\eta, h) \in \mathcal{R}^d$, then

$$P^N\{V(\theta_N) \leq \varepsilon\} \geq 1 - \beta = 1 - \sum_{i=0}^{d-1} \binom{N}{i} \epsilon^i (1-\epsilon)^{N-i}$$

Scenario approach

$$\min_{(\eta,h)} h$$

subject to $l_{\delta^{(i)}}(\eta) \leq h, i = 1, \dots, N$

scenario optimization

Take β, say $\beta = 10^{-7}$. Then,

- $N \geq \frac{1}{\varepsilon}\left(d + log\left(\frac{1}{\beta}\right) + \sqrt{2dlog\left(\frac{1}{\beta}\right)}\right)$ to get a violation ε

Scenario approach

$$\min_{(\eta,h)} h$$

subject to $l_{\delta^{(i)}}(\eta) \leq h, i = 1, \dots, N$

scenario optimization

Take β, say $\beta = 10^{-7}$. Then,

- $N \geq \frac{1}{\varepsilon}\left(d + log\left(\frac{1}{\beta}\right) + \sqrt{2dlog\left(\frac{1}{\beta}\right)}\right)$ to get a violation ε
- If $\bar{N}$ realizations are available (data driven problem), then

$$P^{\bar{N}}\{V(\theta_{\bar{N}}) \leq \bar{\varepsilon}\} \geq 1 - \beta \text{ where } \bar{\varepsilon} = 1 - \sqrt[\bar{N}-d]{\frac{\beta}{\binom{\bar{N}}{d}}}$$

Scenario approach

$$\min_{(\eta,h)} h$$

subject to $l_{\delta^{(i)}}(\eta) \leq h, i = 1, \dots, N$

scenario optimization

Theorem

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upper bound on the cardinality of the support set

Scenario approach

support set

"minimal cardinality subset of the constraints such that by considering only this set of constraints, we obtain the same solution"

Intuition:

all constraints that do not belong to the support set are in a sense redundant → generalization to unseen scenarios

Scenario approach: a posteriori result

Let $\hat{d}$ be the cardinality of the support set for the extracted scenario program, then,

$$P^{\bar{N}}\{V(\theta_{\bar{N}}) \leq \hat{\varepsilon}(\hat{d})\} \geq 1-\beta \text{ where } \hat{\varepsilon}(k) = 1 - \sqrt[\bar{N}-k]{\frac{\beta}{(\hat{d}+1)\binom{\bar{N}}{k}}}$$

Remarks:

- also for non-convex problems
- 'wait and see' a-posteriori result

Centralized stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \text{subject to } x \in \bigcap_{k=1}^{N} \bigcap_{i=1}^{m} X_i(\delta^{(k)}) \quad \longrightarrow \quad x_N^c$$

how does the solution x_N^c generalizes to unseen scenarios?

$$\mathbb{P}\left\{\delta : x_N^c \notin \bigcap_{i=1}^{m} X_i(\delta)\right\}$$

Centralized stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \text{subject to } x \in \bigcap_{k=1}^{N} \bigcap_{i=1}^{m} X_i(\delta^{(k)}) \quad \longrightarrow \quad x_N^c$$

known upper bound for the support set cardinality

- with confidence $1-\beta$ it holds that

$$\mathbb{P}\left\{\delta : x_N^c \notin \bigcap_{i=1}^{m} X_i(\delta)\right\} \leq \varepsilon = 1 - \sqrt[N-d]{\frac{\beta}{\binom{N}{d}}}$$

Centralized stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \text{subject to } x \in \bigcap_{k=1}^{N} \bigcap_{i=1}^{m} X_i(\delta^{(k)}) \longrightarrow x_N^c$$

known upper bound for the support set cardinality

- with confidence $1-\beta$ it holds that

$$\mathbb{P}\left\{\delta : x_N^c \notin \bigcap_{i=1}^{m} X_i(\delta)\right\} \leq \varepsilon = 1 - \sqrt[N-d]{\frac{\beta}{\binom{N}{d}}}$$

- x_N^c can be computed via the proposed distributed algorithm, but scenarios should be available to all agents

Distributed stochastic optimization

- Scenarios (constraints) are private resources
- Each agent has its own N_i scenarios/realizations of δ

$$\delta^{(i,1)}, \delta^{(i,2)}, \ldots, \delta^{(i,N_i)}$$

Distributed stochastic optimization

- Scenarios (constraints) are private resources
- Each agent has its own N_i scenarios/realizations of δ

$$\delta^{(i,1)}, \delta^{(i,2)}, \ldots, \delta^{(i,N_i)}$$

Construct the scenario program

$$\min_x \sum_{i=1}^{m} f_i(x)$$
$$\text{subject to } x \in \bigcap_{i=1}^{m} \left[\bigcap_{k=1}^{N_i} X_i(\delta^{(i,k)}) \right]$$

Distributed stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x)$$
$$\text{subject to } x \in \bigcap_{i=1}^{m} \left[\bigcap_{k=1}^{N_i} X_i(\delta^{(i,k)}) \right] \Longrightarrow x_N^d$$

- fits the distributed set-up with

$$\bigcap_{k=1}^{N_i} X_i(\delta^{(i,k)}) \text{ in place of } X_i$$

- x_N^d can be computed via the proposed distributed algorithm

Distributed stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x)$$

$$\text{subject to } x \in \bigcap_{i=1}^{m} \left[\bigcap_{k=1}^{N_i} X_i(\delta^{(i,k)}) \right] \longrightarrow x_N^d$$

$$\mathbb{P}\left\{ \delta : x_N^d \notin \bigcap_{i=1}^{m} X_i(\delta) \right\} \leq \; ?$$

Distributed stochastic optimization

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solution to the
distributed problem,
with local scenarios

Distributed stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \text{subject to } x \in \bigcap_{i=1}^{m} \left[\bigcap_{k=1}^{N_i} X_i(\delta^{(i,k)}) \right] \longrightarrow x_N^d$$

$$\mathbb{P}\left\{ \delta : x_N^d \notin \bigcap_{i=1}^{m} X_i(\delta) \right\} \leq \; ?$$

solution to the distributed problem, with local scenarios

constraint of the centralized problem, with global scenarios

Distributed stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \text{subject to } x \in \bigcap_{k=1}^{N} \bigcap_{i=1}^{m} X_i(\delta^{(k)}) \longrightarrow x_N^d$$

Theorem (extensions of scenario theory to distributed optimization)

Fix β and choose β_i such that $\sum_{i=1}^{m} \beta_i = \beta$. Then, with confidence 1-β it holds that

$$\mathbb{P}\left\{ \delta : x_N^d \notin \bigcap_{i=1}^{m} X_i(\delta) \right\} \leq \sum_{i=1}^{m} \varepsilon_i(\hat{d}_i)$$

a-posteriori bound

Distributed stochastic optimization

$$\min_x \sum_{i=1}^{m} f_i(x) \quad \text{subject to } x \in \bigcap_{k=1}^{N} \bigcap_{i=1}^{m} X_i(\delta^{(k)}) \longrightarrow x_N^d$$

Theorem (extensions of scenario theory to distributed optimization)

Fix β and choose β_i such that $\sum_{i=1}^{m} \beta_i = \beta$. Then, with confidence 1-β it holds that

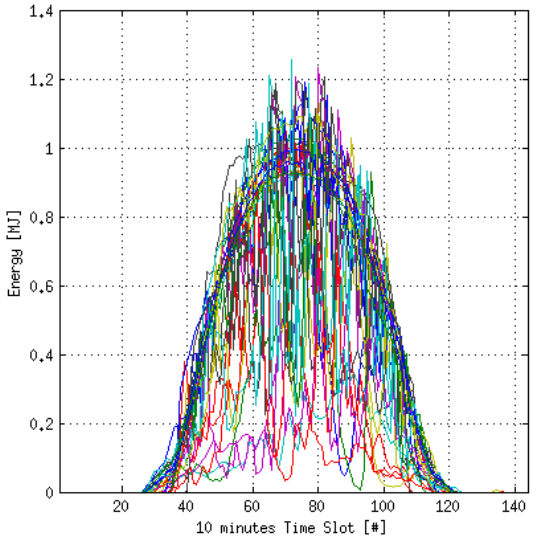
$$\mathbb{P}\left\{\delta : x_N^d \notin \bigcap_{i=1}^{m} X_i(\delta)\right\} \leq \sum_{i=1}^{m} \varepsilon_i(\hat{d}_i) \leq \varepsilon = \max_{\{d_i \in \mathbb{N}_+\}_{i=1}^{m}} \sum_{i=1}^{m} \varepsilon_i(d_i) \quad \text{subject to } \sum_{i=1}^{m} d_i \leq d$$

a-priori bound

Conclusions

- New results on distributed data-driven optimization
- Extension of the scenario approach to distributed convex optimization
- Still much work need to be done to quantify achievable performance and to obtain receding horizon implementation

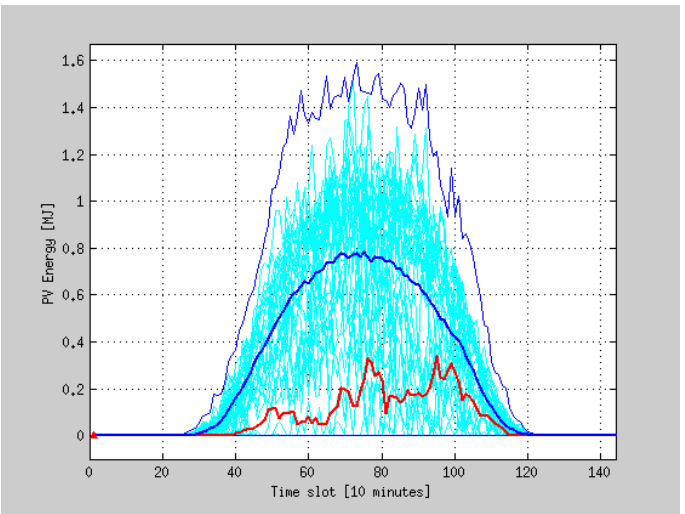
Uncertainty description “a-priori”



30 realizations of the energy produced by a photovoltaic panel installation

[courtesy of GE Global Research Europe, Munich]

Receding horizon implementation



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H2020, 2015-2018

Thank you for your attention!