Metamaterial-Based Foundation System Endowed with Non-Linear Oscillators for the Protection of Fuel Storage Tanks

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Abstract

In recent years the advance of Metamaterials has extended into the field of seismic engineering. Many novel concepts for vibration mitigation have been proposed ranging from Metabarriers embedded in the soil [1], over supporting structures with negative Poisson ratios [2], to Foundations endowed with local resonators [3]. In this work, we present the development of a tank-foundation based on locally resonant metamaterials with non-linear oscillators. These non-linear oscillators are desired to offer a negative stiffness [4] and are therefore referred to as negative stiffness elements (NSEs). To give the reader a better idea of the mechanism, the structure is represented in Figure 1.

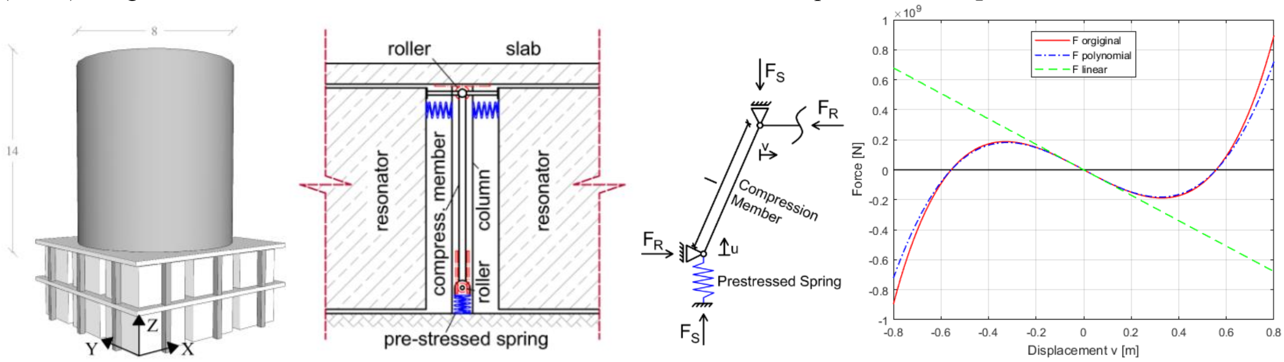

Figure 1. System design: (a) view of the complete system; (b) NSE mechanism; (c) force equilibrium; (d) force displacement path of the NSE for the resonator.

Here, the basic idea is to implement resonators into a foundation, which are in terms endowed with the NSEs. The NSEs essentially consist of a prestressed compression member in a snap through position that is held in equilibrium by the resonators and their respective springs. When establishing the force equilibrium around the compression member, as depicted in Figure 1(c), the force displacement diagram can be obtained with,

$$F_R(v) = \frac{v(-P + \mathrm{k}_p(l - \sqrt{l^2 - v^2}))}{\sqrt{l^2 - v^2}} \tag{1}$$

and is shown in Figure 1(d). In order to estimate the effect that the non-linearity has on the metamaterial, the harmonic balance method (HBM) will be used to calculate the band gaps. More precisely, a complex Fourier series will be employed in its more general form, where both complex conjugates of the amplitudes are considered. After applying this and the HBM to the system, an analytic equation for the dispersion relation can be found that is dependent on the non-linearity of the system. The formulation of the Floquet-Bloch boundary condition under consideration of the HBM reads,

$$u_{j\pm 1}(t) = \sum_{k}^{n} (B_k e^{ik\omega t} + \bar{B}_k e^{-ik\omega t})\, e^{\pm i q_k} \tag{2}$$

For the application of the HBM, it is necessary to define the number of harmonics that has to be employed for a good approximation. Since it is unknown how many harmonics are necessary for the formulation in (1), we use a polynomial approximation of the 3rd order for the force displacement path. The third order polynomial can be well approximated with the 1st and 3rd harmonic, can be seen in Figure 1(d) as a blue dash-dot line and reads,

$$F_R(v) = av + bv^3; \qquad a = -\frac{P}{l}; \qquad b = \frac{k_p}{l(2k_p l - P)} \tag{3}$$

Another neat thing about the polynomial approximation is that it makes a clear distinction between the linear and non-linear part of the NSE. In (3), a clearly depicts the linear initial stiffness, while b represents the hardening of the mechanism. It is now possible to apply (2) and (3) onto the equations of motion of the typical unit cell, represented in Figure 2 (a), and derive the non-linear dispersion equation of the system for its first harmonic as,

$$Cos(q_1) = \frac{2k_1k_2 - k_2m_1\omega^2 - 2k_1m_2\omega^2 - k_2m_2\omega^2 + m_1m_2\omega^4 + a(2k_1 + 2k_2 - (m_1+m_2)\omega^2) + 3bB_1\overline{B}_1(2k_1 + 2k_2 - (m_1+m_2)\omega^2)}{2(a(k_1+k_2) + 3bB_1\overline{B}_1(k_1+k_2) + k_1(k_2 - m_2\omega^2))} \qquad (4)$$

The non-linearity of the system is clearly determined by the term $bB_1\overline{B}_1$, where b represents the non-linear part of the polynomial fit, while $B_1\overline{B}_1$ denote the complex wave amplitude of the Fourier spectrum and its complex conjugate, respectively. For the sake of demonstration, we fixed the system in terms of geometry and therefore also its non-linear behavior b and kept only the amplitude of the travelling wave $B_1\overline{B}_1$ as a non-linear parameter. The resulting dispersion curves can be seen in Figure 2 (b) and show that for an increase in non-linearity the band-gap shifts towards higher frequencies. This makes sense since the negative stiffness provides a softening effect in the system, which reduces with an increase in amplitude, also see Figure 1 (d). This system will, in future investigations, be coupled with a superstructure and subjected to real seismic records. Therefore, the complete system will be optimized in the frequency domain and tested on its performance with non-linear time history analyses.

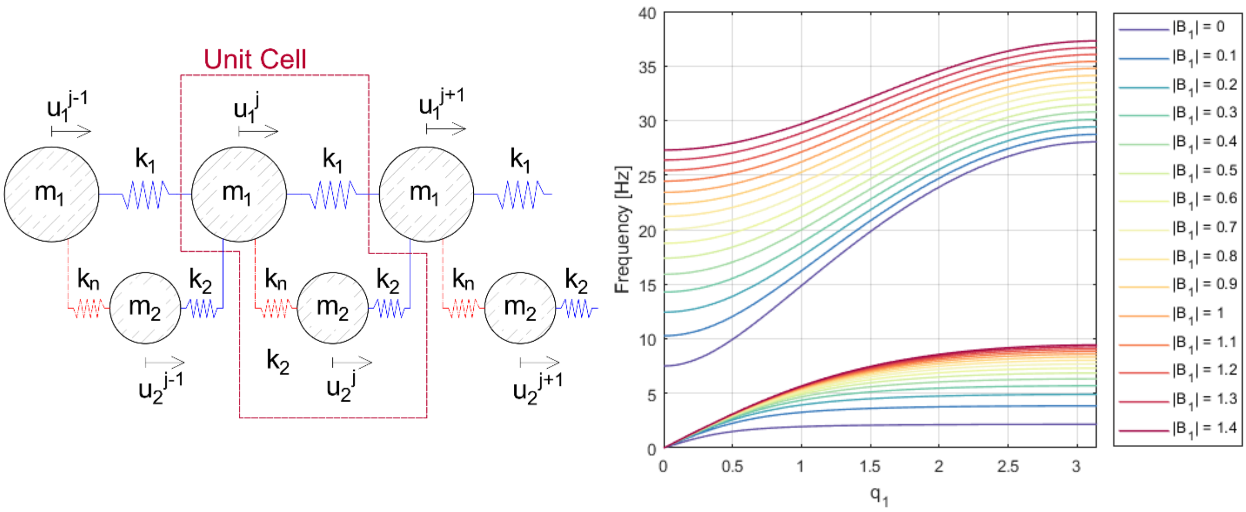

Figure 2. Band-gap analysis: (a) unit cell depiction; (b) amplitude dependent dispersion curves.

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References

[1] Palermo A., Krödel S., Marzani A., Daraio C., 2016, *Engineered metabarrier as schield from seismic surface waves*. Scientific Reports 6, DOI: 10.1038/srep39356

[2] Ungureanu B., Achaoui Y., Enoch S., Brule S., Guenneau S., 2015, *Auxectic-like metamaterials as novel earthquake protections*. EPJ Applied Metamaterials 2(17), DOI: 10.1051/epjam/2016001

[3] La Salandra V., Wenzel M., Bursi O.S., Carta G., Movchan A.B., 2015, *Conception of a 3D Metamaterial-Based Foundation for Static and Seismic Protection of Fuel Storage Tanks.* Front. Mater. 4:30. DOI: 10.3389/fmats.2017.00030

[4] Antoniadis I., Chronopoulos D., Spitas V., Koulocheris D., (2015), *Hyper-damping properties of a stiff and stable linear oscillator with a negative stiffness element.* Journal of Sound and Vibration 346, 37-52. DOI: 10.1016/j.jsv.2015.02.028